Towards AdS/CFT, with black holes

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The result (3.5.4) refers to a black hole in asymptotically flat space. However, as we have argued, the asymptotic density of states in a quantum theory of gravity will depend on the value of the cosmological constant.

Static black holes in anti-deSitter spacetime are described by

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_{D-2}\,, \tag{3.7.1}$$

with

$$V(r) = 1 - \frac{\mu}{r^{D-3}} + \frac{r^2}{L_{AdS}^2} \tag{3.7.2}$$

where μ is the mass parameter, $M \propto \mu/G$, and $L_{AdS} \propto 1/\sqrt{-\Lambda}$ is the cosmological radius. Black hole horizons are located at radii where $V(r) = 0$. When $\mu \ll L_{AdS}^{D-3}$ the horizon radius is $r_H \simeq \mu^{1/(D-3)}$, much smaller than L_{AdS} and the black hole hardly feels the presence of the cosmological constant: it is much like the Schwarzschild solution in AF space.

However, we are more interested in what happens when the energy is large, $\mu \gg L_{AdS}^{D-3}$. In this case[52] we can neglect the constant '1' in $V(r)$ and find the horizon from

$$\frac{\mu}{r_H^{D-3}} \simeq \frac{r_H^2}{L_{AdS}^2} \tag{3.7.3}$$

i.e.,

$$r_H^{D-1} \simeq \mu L_{AdS}^2 \sim G E L_{AdS}^2\,, \tag{3.7.4}$$

or[53]

$$\left(\frac{r_H}{L_{Planck}}\right)^{D-1} \sim \frac{L_{AdS}}{L_{Planck}} E L_{AdS}\,. \tag{3.7.5}$$

The entropy is

$$S_{BH} = \frac{\Omega_{D-2}}{4}\left(\frac{r_H}{L_{Planck}}\right)^{D-2} \sim \left(\frac{L_{AdS}}{L_{Planck}}\right)^{\frac{D-2}{D-1}} (E L_{AdS})^{\frac{D-2}{D-1}}\,. \tag{3.7.6}$$

If we compare this to (3.5.3) we see that the scaling with the energy differs from that of a CFT in D spacetime dimensions. However, it is the same as that of a $(D-1)$-dimensional CFT on a sphere of radius $R \sim L_{AdS}$, with[54]

$$c \sim \left(\frac{L_{AdS}}{L_{Planck}}\right)^{\frac{D-2}{D-1}}. \tag{3.7.7}$$

The conclusion is that AdS$_D$ gravity may admit a description in terms of a CFT$_{D-1}$ with a large number of fields of the order of c in (3.7.7). This must be large if the curvature radius of the spacetime L_{AdS} is to be much larger than the Planck length, which is required for the semiclassical validity of the GR description. Observe that this number of fields does depend on the cosmological constant. So, in the fundamental theory, the latter will be quantized, and moreover, different values of Λ will correspond to different theories.

[53] Recall $G\hbar = L_{Planck}^{D-2}$.

[54] Observe that this factor is extracted after factorizing out all the dependence on the energy. The microscopic Planck scale, which could not be eliminated from (3.5.4), has now been transferred to the parameter c via the introduction of a macroscopic scale L_{AdS}.