Gravity is not Wilsonian

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Wilsonian QFT. Conventional (textbook, Lagrangian) quantum field theories are based on the idea that one can separate the propagation of the field from its interactions. The propagation is controlled by a free theory, and this is what dominates the short-distance structure of the field[49]. Interactions are then added on as perturbations. Wilson generalized this framework to include theories in which the short-distance structure need not be dominated by a free theory, but rather one in which interactions may be strong. If there is a good limit to the theory at asymptotically high energies, this should be a Conformal Field Theory (CFT), *i.e.*, one that is invariant under (local) changes of the energy scale[50]. The analogue of the perturbative interactions of a free theory is now played by perturbations of the CFT by operators that may become important (relevant) at lower energies, but whose effects do not grow, or more typically die off, at higher energies.

Locality plays a crucial role here. One assumes that in order to define the CFT, one has to look at the short-distance structure of the theory. It is there where the fundamental degrees of freedom are manifest. The long distance features of the *state* of the field are not fundamental, but rather properties of *specific solutions* of the theory.

One consequence of this locality is that the total number of degrees of freedom should be an extensive quantity, growing like the volume of the region one is considering. Take the theory to be defined on a spatial sphere S^{D-1} of radius R. In order to see how the spectrum of states grows with the energy, we can imagine putting the system at temperature T, and measuring its total entropy and energy. Since the entropy is dimensionless but must scale like the spatial volume, dimensional analysis dictates that

$$S_{CFT} = a(RT)^{D-1}\,, \tag{3.5.1}$$

and thus the energy

$$E = a\frac{D-1}{D}R^{D-1}T^D\,. \tag{3.5.2}$$

Then

$$S_{CFT} = c(ER)^{\frac{D-1}{D}} \tag{3.5.3}$$

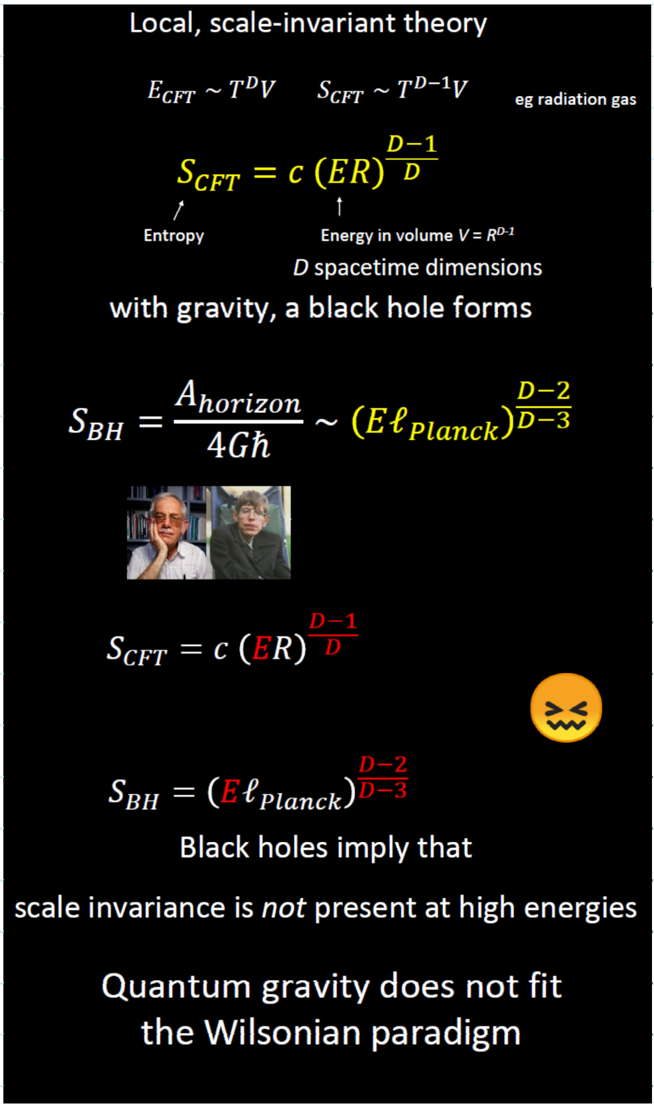
The number c (and a, which is fixed by it) is theory-dependent, and provides a measure of the number of local fields (species of degrees of freedom) in the theory.

From our considerations above, the possibility that a QFT of gravity can be defined in this way would seem to require that

> the entropy that characterizes the density of states in a region of size R should scale with energy like (3.5.3), independently of what the state of the field is at long distances.

In addition, since the CFT fixed point dominates physics when all kinematic invariants are large, its asymptotic scale invariance dictates that

> correlation functions should depend only on powers of ratios of the ingoing and outgoing momenta.



Non-Wilsonian character of GR. *What we know about semiclassical black holes tells us that neither of these two properties is satisfied by GR.* Already the thought scattering experiments that we discussed above have revealed that scattering amplitudes are not hard: for '2 → 2' scattering, the amplitudes are not power laws, but are exponentially suppressed by $e^{-S_{BH}} \sim e^{-E^2/E^2_{Planck}}$.[51] The probabilities to emit quanta of a given energy ω are $\sim e^{-\omega E/E^2_{Planck}}$.

Moreover the long-distance properties of the spacetime are crucial to the outcome of the scattering event: at higher and higher energies, the black hole that one forms gets larger and larger. Thus, how its area (entropy) grows with mass, and how the black hole decays, are going to be sensitive to what the spacetime is like at large length scales.

If we try to localize energy into a region of size R, we know that in GR eventually we will form a black hole. If the spacetime is asymptotically flat, then the density of states that we get does not scale like (an exponential of) the volume R^{D-1}, but rather like the area R^{D-2}. Thus we will not obtain (3.5.3). Instead, the Schwarzschild-type black hole that one creates has

$$S_{BH} = b\left(\frac{E}{E_{Planck}}\right)^{\frac{D-2}{D-3}} \tag{3.5.4}$$

(again b is a pure number). The different scaling with energy indicates that the asymptotic spectrum of states is not that of a CFT.