

Breakdown of quantum GenRel

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Growth of gravitational interaction. The perturbative effective theory of GR breaks down at energies $E \sim M_{\text{Planck}}$. One way to see this is that scattering amplitudes grow too large. The theory is strongly coupled. Perturbative unitarity bounds are violated.³⁴ The reason is that gravitons couple to energy, and therefore the scattering amplitudes grow unbounded at very high energies. In fact, gravity becomes the dominant interaction at Planckian energies. In more detail, consider the $2 \rightarrow 2$ scattering at large center-of-mass energy, when mediated by exchange of particles of different spin³⁵:

³⁵We characterize it with the Mandelstam kinematic invariants, $s = -(p_1 + p_2)^2$ for the center of mass energy squared, $t = -(p_1 - p_3)^2$ for (minus) the square of the exchanged momentum.

- For scalar interactions (*e.g.*, ϕ^3), the vertices do not depend on energy, and thus the scattering amplitude behaves like $A_0(s, t) \sim s^0/(-t)$.
- Vectors A_μ couple to currents (*i.e.*, to velocities) so their vertices carry factors of $p^\mu \sim \sqrt{s}$, and thus scattering amplitudes grow like $A_1(s, t) \sim s/(-t)$.
- Tensors $h_{\mu\nu}$ (gravitons) couple to stress-energy $p^\mu p^\nu \sim s$ (this is visible in the two-derivative vertex (3.1.5)), so the scattering amplitudes grows like $A_2(s, t) \sim s^2/(-t)$,³⁶ and the cross section like $\sigma \sim s^2$.³⁷

In short: the effective dimensionless coupling for gravity, Gs , runs with the energy as a power and eventually overwhelms any other interaction. Gravity becomes strongly coupled when $Gs > 1$, and the perturbative expansion in the effective theory of quantum GR is no longer sensible.

Enter new UV degrees of freedom? The conventional QFT wisdom says that at this characteristic scale new degrees of freedom must enter that unitarize the theory. For instance, the Fermi theory of weak interactions remains very good at energies $\lesssim O(100 \text{ GeV})$ but the scattering amplitudes violate unitarity above this energy. Then one resorts to the electroweak theory which resolves the four-fermion, current-current interaction vertex into a 2 fermion-2 fermion interaction mediated by W and Z gauge bosons with masses $O(100 \text{ GeV})$. In this case

we introduce new degrees of freedom in order to *resolve the interactions*³⁹ but the other degrees of freedom (leptons, quarks, photons) are still part of the high-energy theory. In other instances *the low-energy degrees of freedom are themselves resolved* into more fundamental ones, and thus the interactions are smeared, too. This is the case for QCD, for which at energies below the chiral-symmetry-breaking scale the physics is described using a non-linear sigma model for pions, while at high energies these are seen to be quark-antiquark bound states. The fundamental theory involves variables (quarks and gluons) that are invisible at low energies. Observe that in this case, similarly to the case of gravity, it is the derivative nature of the couplings of (Goldstone) pions that brings in the bad high energy behavior.

Thus one may expect that gravitons, or at least their interactions, are modified by the appearance of new, short-distance degrees of freedom. Indeed, string theory would seem to realize this goal by resolving *both the particles and their interactions* into extended string-like objects that merge and split forming continuous worldsheets. Actually, if the coupling of closed strings is weak, $g_s \ll 1$, they modify the scattering even before the Planck scale is reached. The stringy nature of particles begins to be visible (in the form of low-lying massive excitations of the string) at a scale $M_{\text{string}} \sim g_s M_{\text{Planck}} \ll M_{\text{Planck}}$.

Scalar field interactions do not depend on energy

$$\lambda \phi^4$$

Vector field interactions (electromagnetism) *grow* with velocity (currents)

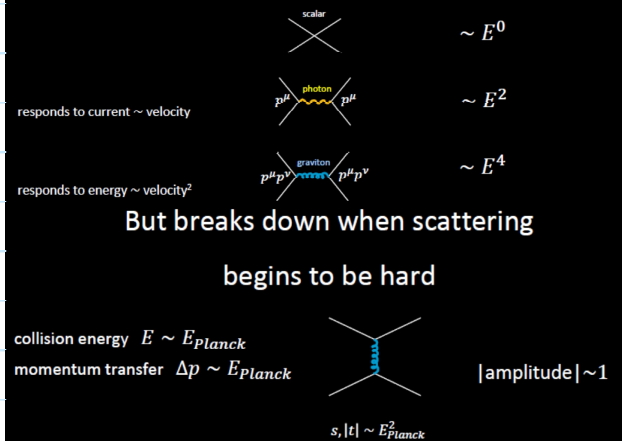
$$j^\mu A_\mu \quad j^\mu = ev^\mu$$

Gravity responds to energy

$$T^{\mu\nu} h_{\mu\nu} \quad T^{\mu\nu} = m v^\mu v^\nu$$

Gravity

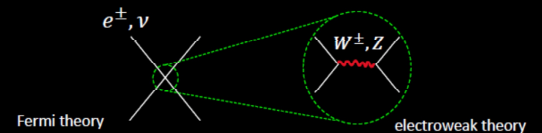
The weakest force, the strongest force



Enter new physics?

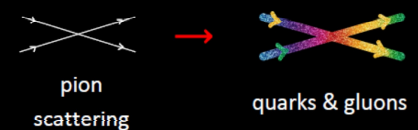
Weak example

“Resolve” interactions with new massive particles



Strong example

Resolve interactions *and* particles with more fundamental ones



Gravity seems more similar to the second option



strings