AdS/CFT realized

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The previous ideas have been realized in several cases. The most famous one is the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence between the $\mathcal{N}=4$ supersymmetric Yang-Mills theory with group $SU(N)$ and type IIB supergravity in $\mathrm{AdS}_5 \times S^5$. The S^5 in the latter is a compact space and does not add to the dimensionality of the boundary, but instead realizes the conformal scalar fields of the Yang-Mills theory. The number of local fields is $c \sim N^2$ and thus the rank of the gauge group must be large in order for the gravitational description to be valid. One can in fact go beyond the gravitational regime in the AdS side of the correspondence, and consider type IIB string theory in this spacetime.

Eq. (7.42), with $D = 5$, contains the 5D Planck length $L_{Planck(5)}$. This is related to the 10D Planck length in $\mathrm{AdS}_5 \times S^5$ as

$$\left(L_{Planck(10)}\right)^8 = \left(L_{Planck(5)}\right)^3 \left(L_{S^5}\right)^5 = \left(L_{Planck(5)}\right)^3 \left(L_{AdS}\right)^5 \tag{7.43}$$

so

$$c \sim \left(\frac{L_{AdS}}{L_{Planck(5)}}\right)^{3/4} \sim \left(\frac{L_{AdS}}{L_{Planck(10)}}\right)^2 . \tag{7.44}$$

Since $c \sim N^2$, the AdS radius measured in Planck units is

$$\frac{L_{AdS}}{L_{Planck(10)}} \sim N^{1/4} . \tag{7.45}$$

Generically, a holographic theory need not have any more dimensionless parameters than c, but the SYM theory does have another one: the gauge coupling constant (a continuous modulus) g_{YM}, or the 't Hooft coupling $\lambda = g_{YM}^2 N$. This introduces another parameter in the gravitational side, namely the string coupling constant

$$g_s \sim g_{YM}^2 \sim \frac{\lambda}{N} , \tag{7.46}$$

and since $L^8_{Planck(10)} \sim g_s^2 \ell^8_{string}$,

$$\frac{L_{AdS}}{\ell_{string}} \sim \lambda^{1/4}\,. \tag{7.47}$$

String effects are small for large λ.

7.2 Some remarks on the correspondence

Planck and string scales from a scale-free CFT. The parameters for the microscopic theory are only N and g_{YM}. The Planck length has not in any way been introduced as a fundamental minimal length. Neither has the string length. So this quantum theory of gravity *does not contain any dimensionful parameter, but instead has two dimensionless parameters.* It is interesting to contrast this to the emphasis often made on the seemingly appealing fact that string theory does not contain any fundamental dimensionless adjustable parameters, but only has a dimensionful one (the string tension). In the quantum theory of gravity in AdS the absence of a fundamental parameter for the Planck scale is possible because the theory has a cosmological constant.

In other words, the CFT only contains pure numbers, not scales. If these pure numbers (*e.g.*, dimensions of operators) are to be translated into magnitudes of a dual theory that necessarily involves dimensionful scales, such as a string theory or a quantum gravity theory, then we conclude that the latter theories must be defined in a spacetime that contains a length scale. We choose N and g_{YM} to be large in order to have large parametric separations in the mass spectrum of the dual theories.

String theory has also provided other quantum formulations of gravitational theories that are presumably complete, and which are also holographic in nature: M(atrix) theory, little string theories, and dualities to theories of membranes or fivebranes. All these are, to varying degrees, less well understood than the previous ones.

Matrix quantum mechanics is very interesting since it may provide large classes of quantum theories of gravity that defy traditional assumptions. In the large N limit, these models present some features of gravity and black holes (large degeneracies, scrambling and chaos). If the matrices are endowed with a vector index with symmetry $SO(d)$, then they may yield a quantum theory of gravity in $d+1$ or $d+2$ dimensions. Supersymmetry does not seem to be required for finiteness, which is automatic because the theory is simply quantum mechanics. So there appears the possibility that finite quantum gravity exists in any dimension, and without supersymmetry.

Before reaching this conclusion, we must bear in mind that there are at least two other conditions that seem necessary to claim that we have a good holographic quantum theory of gravity: stability (boundedness-below) of the spectrum, and a large hierarchy between a few operators of low dimension and other operators of large dimension. For these two purposes, supersymmetry may be very helpful, but it is unclear whether it is indispensable.