The Black Hole Guide to the Quantum Theories of Gravity

Roberto Emparan

Institució Catalana de Recerca i Estudis Avançats (ICREA)

Passeig Lluís Companys 23, E-08010 Barcelona, Spain

Departament de Física Quàntica i Astrofísica, Institut de Ciències del Cosmos,

Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain

One sometimes hears that "we don't know anything about quantum gravity". The purpose of this lecture is to argue that, even if certainly we do not understand everything about quantum theories of gravity, we have learned some important features they must possess—and we do have an explicit complete, non-perturbative formulation of a quantum theory of gravity that incorporates them.

This is largely a consequence of taking at face value what we know about the properties of black holes, at the classical level and from their thermodynamical behavior. Following this lead, we arrive at the following conclusions:

- Gravity is not a conventional (Wilsonian) quantum field theory, with a microscopic formulation in terms of local degrees of freedom. Instead, the fundamental formulation of a quantum theory of gravity necessarily involves the specification of spacetime asymptotics (this can be regarded as a manifestation of the UV/IR connection).

- Quantum theories of gravity are holographic, *i.e.,* defined in terms of a (non-gravitational) quantum theory that lives in the asymptotic boundary of spacetime.

- One should not talk about "*the* theory of quantum gravity", but rather of "quantum theor*ies* (or models) of gravity": there are probably many quantum theories, with different microscopic degrees of freedom (*e.g.,* corresponding to different asymptotics, different spacetime dimensions, or different bulk fields) that in some limit admit an approximate semiclassical description in terms of gravitational variables and dynamics.

Admittedly, the arguments that lead to these (interrelated) conclusions make a number of more or less explicit assumptions, and therefore there is the logical possibility that consistent quantum theories of gravity exist which do not conform to them. If this were the case, these alternative theories should explain what is wrong with the arguments based on apparently robust properties of black holes.[1] It must be borne in mind that these typically involve non-perturbative features of the gravitational theory, so they may not be apparent if one only has a perturbative definition of the theory.

A main reason for believing that these ideas are correct is that they are magnificently realized in our most successful formulation of a quantum theory of gravity, the AdS/CFT correspondence. Indeed we will progressively build the argument so that it naturally leads to this correspondence.[2]

We assume that the search for the fundamental formulation of the theory is most helpfully guided by an investigation of its properties at very high energies. More specifically, the properties of the asymptotic spectrum of the theory at high energies tell us about its fundamental degrees of freedom. Thus we will study the behavior of gravitational scattering at progressively higher energies.

The argument in a nutshell

In the following sections the argument for holography is developed in some detail, aimed at the beginning graduate level. Readers with more expertise will probably find this exposition way too morose (I do feel the presentation is too long: I haven't had the time to make it shorter). To help them skip along, I summarize here the main lines of the argument:

GR can be quantized as an effective theory for scattering at $E \ll E_{Planck}$. The fluctuations of the gravitational field around a given background can be quantized using conventional QFT methods. The theory is not renormalizable perturbatively, but it can be treated as a low-energy effective field theory that is predictive with precision within a power of E/E_{Planck} — but, as emphasized below, it runs into serious difficulties when trying to describe phenomena related to black hole evaporation.

GR can be used to describe scattering at $E \gg E_{Planck}$. The unbounded growth of scattering amplitudes at high energies, due to gravity's coupling to stress-energy, leads to a breakdown of perturbative unitarity. However, this same effect points to the importance of non-perturbative effects, whose archetype is the formation of black holes, dominated by classical dynamics. The outside observer is causally shielded from the trans-Planckian regime and can describe the physics using low-energy classical field theory, now in a non-perturbative regime.

[1] E.g., there may exist non-holographic but consistent quantum theories of gravity that do not have black holes with a large entropy, and Liouville theory is an example (it does not have propagating degrees of freedom either). But these theories may be very simple and of limited interest.

[2] I do not claim any originality, other than possibly of aspects of the presentation, in the ideas exposed below. Others have developed these arguments earlier, and in a deeper manner — see in particular T. Banks, `hep-th/0306074, 1007.4001`. My main aim is to be pedagogical, rather than as rigorous or complete as one could be.

Gravity is a non-Wilsonian theory. The appearance of a large classical length scale at high energies implies that the degrees of freedom at high energies are sensitive to the long-distance properties (asymptotics) of the field. This goes against the usual decoupling of energy scales in Wilsonian QFT. Correlation functions at very high energies do not behave as if they were dominated by a conformal fixed point. Moreover, the problems with unitarity in black hole evolution indicate that, despite the two previous items, something very subtle is amiss in the framework of local QFT, including GR, when trying to describe quantum gravitational effects even at large length scales (or very long times). Semiclassical quantum General Relativity is not good enough.

Gravity is a holographic theory. The growth with area of the maximum entropy contained within a region indicates that the fundamental degrees of freedom are localized at the boundary of spacetime. In fact, already the classical theory of GR gives indications of this: it does not admit strictly local, diffeo-invariant observables. Instead, quasi-local observables, defined on codimension-1 surfaces, are well defined. In the case of AdS_D asymptotics, the microscopic theory is a conformal field theory in $D-1$ dimensions.

In the first two items I refer to the semiclassical theory of gravity described at leading order by Einstein's theory. When I say that the theory is good, I mean that it appears to give a good approximation to many observables, although we will see that it misses crucial subtleties when quantum effects on horizons are present over long times. As mentioned above, it seems extremely difficult, if not impossible, to recover unitarity in a local quantum field theory based on gravitational variables.

In the last two items 'Gravity' refers to the full quantum theory that contains GR in a semiclassical limit. The gravitational field is presumably not a variable in the fundamental formulation.

In the next section we start from the regime of sub-Planckian energies, where conventional ideas about quantum field theory still apply. This is fairly standard lore, which students may already be familiar with, but we review it for completeness[3]. It is only when we push beyond this regime that black holes make their appearance and force a departure from the usual picture.

1 Sub-Planckian regime: quantum General Relativity as an effective field theory

It is widely accepted that at energies below the Planck scale one can treat quantum General Relativity as an effective quantum field theory. One can extract those finite quantum-gravitational effects that are due to low-frequency gravitons running in loops, since these are not sensitive to the ultraviolet behavior of the theory.

[3] My presentation is largely based on reviews by J. Donoghue, *e.g.,* `gr-qc/9512024`.

Perturbative non-renormalizability of quantum GR. Consider the Einstein-Hilbert theory

$$I = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}R\,, \qquad \kappa^2 = 32\pi G \tag{1.1}$$

and study small fluctuations around Minkowski space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,. \tag{1.2}$$

Expanding in powers of κ,

$$\frac{2}{\kappa^2}\sqrt{-g}R = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \dots \tag{1.3}$$

where, schematically (suppressing Lorentz indices and numerical factors),

$$\mathcal{L}^{(2)} \sim \partial h\partial h \tag{1.4}$$

i.e., terms quadratic in h which yield the propagator of the graviton, while the cubic terms

$$\mathcal{L}^{(3)} \sim \kappa h\partial h\partial h \tag{1.5}$$

account for the leading self-interaction of gravitons. Observe that this vertex contains two derivatives, *i.e.,* two factors of momenta. In the expansion there appear vertices $\mathcal{L}^{(n)}$ involving an arbitrary number of gravitons.

If we expand instead around a non-trivial background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \tag{1.6}$$

then

$$\frac{2}{\kappa^2}\sqrt{-g}R = \sqrt{-\bar{g}}\left(\frac{2}{\kappa^2}\bar{R} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \dots\right) \tag{1.7}$$

where

$$\mathcal{L}^{(1)} = \frac{h^{\mu\nu}}{\kappa}\left(\bar{g}_{\mu\nu}\bar{R} - 2\bar{R}_{\mu\nu}\right) \sim \frac{1}{\kappa}\bar{R}h\,, \tag{1.8}$$

$$\mathcal{L}^{(2)} \sim \nabla h\nabla h + \bar{R}h^2\,, \tag{1.9}$$

$$\mathcal{L}^{(3)} \sim \kappa h\nabla h\nabla h + \kappa\bar{R}h^3 + \dots\,, \tag{1.10}$$

This theory is not renormalizable perturbatively. When one computes loop diagrams, there are divergences that cannot be absorbed by a renormalization of the coupling κ. Veltman and 't Hooft showed that at one loop the divergences (from gravitons only) are

$$\mathcal{L}^{(div)}_{1-loop} = \frac{1}{8\pi^2\epsilon}\left(\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right)\,, \tag{1.11}$$

where $\epsilon = 4 - D$ is the parameter in dimensional regularization. Renormalizing this divergence requires the introduction of higher-order terms in the bare action. When expanding around vacuum, $\bar{R}_{\mu\nu} = 0$, this divergence does not appear so the theory is finite at one loop. However, in this case trouble reappears at two loops, where Goroff and Sagnotti found

$$\mathcal{L}^{(div)}_{2-loop} = \frac{209}{2880}\frac{\kappa^2}{16\pi^2\epsilon}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\eta\sigma}R^{\eta\sigma}{}_{\alpha\beta}\,, \tag{1.12}$$

which again cannot be renormalized within the original action.

Quantum General Relativity as an effective theory. The theory nevertheless is sensible as a low-energy effective field theory, and can adequately describe scattering processes in which all kinematic invariants remain below the Planck energy, at least within a precision bounded by a power of E/E_{Planck}.

In a low-energy effective theory one considers all terms in the action that are consistent with the symmetries of the system, so[4]

$$I = \int d^4x\sqrt{-g}\left(\Lambda + \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + \ldots\right) . \tag{1.13}$$

The value of the cosmological constant term Λ is a notorious problem that we do not want to deal with now, and we set it to $\Lambda \approx 0$ on observational grounds ($\Lambda \ll E^4_{Planck}$).

Renormalization of the one-loop divergences in (1.11) can be achieved by

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2}\frac{1}{\epsilon}\,, \qquad c_2^{(r)} = c_2 + \frac{7}{160\pi^2}\frac{1}{\epsilon}\,. \tag{1.14}$$

The $c_i^{(r)}$ are finite and must be determined by experiment: this is the conventional "lack of predictivity" in non-renormalizable theories. However, these higher-order terms naturally give small corrections at long distances, so the theory remains predictive within a definite range of precision. If the coefficients are known experimentally, the precision increases by a factor of $(E/E_{Planck})^2$.

To see this point a little more explicitly, the leading correction to GR is schematically of the form

$$\mathcal{L} = \frac{2}{\kappa^2}R + cR^2\,. \tag{1.15}$$

We linearize the theory (say in TTF gauge) to leading order, neglecting self-interaction, to obtain the, again schematic, equation of motion (with stress tensor T)

$$\Box h + \kappa^2 c\Box^2 h = \kappa^2 T\,. \tag{1.16}$$

Inverting to find the propagator,

$$\begin{aligned} G(x) &= \int \frac{d^4q}{(2\pi)^4}\frac{e^{-iqx}}{q^2+\kappa^2 cq^4} \\ &= \int \frac{d^4q}{(2\pi)^4}\left(\frac{1}{q^2} - \frac{1}{q^2+1/(\kappa^2c)}\right)e^{-iqx}\,. \end{aligned} \tag{1.17}$$

The first term corresponds to the massless graviton propagator. The second reflects the presence in the spectrum of a massive particle[5], with mass $m = 1/\sqrt{\kappa^2 c}$. So the static interaction mediated by exchange of these excitations would seem to yield a potential of the form

$$V(r) = -Gm_1m_2\left(\frac{1}{r} - \frac{e^{-r/\sqrt{\kappa^2c}}}{r}\right)\,. \tag{1.18}$$

Note, however, that the mass of this additional particle is naturally (with c of order one[6]) at

[4] In 4D we do not need to introduce $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$ since through the topologically-invariant Gauss-Bonnet term it can be absorbed in redefinitions of c_1 and c_2.

[5] The minus sign in front of the propagator indicates that this is a ghost with negative kinetic energy. Observe in (1.18) that it gives rise to a repulsive effect.

[6] Given the smallness of the Planck length, actual experimental bounds are almost ridiculously weak, $|c_i| < 10^{74}$.

the Planck scale. In the effective theory we should remain consistently at lower energies, and thus expand the propagator as

$$\frac{1}{q^2+\kappa^2 c q^4} \simeq \frac{1}{q^2} - \kappa^2 c\,, \tag{1.19}$$

so now only the graviton propagates,

$$G(x) = \int \frac{d^4q}{(2\pi)^4}\left(\frac{1}{q^2} - \kappa^2 c\right) e^{-iqx}\,, \tag{1.20}$$

and the additional term yields a contact interaction in the static potential,

$$V(r) = -Gm_1m_2\left(\frac{1}{r} - \#Gc\delta^{(3)}(x)\right)\,. \tag{1.21}$$

Thus the R^2 terms, even if they must appear in the Lagrangian, have effects that are naturally suppressed in calculations that involve large ($> L_{Planck}$) distances.

On the other hand, one can compute the one-loop contributions to the scattering amplitude $\mathcal{A}(q)$ in the Einstein-Hilbert theory. These include loop corrections to graviton exchange and vertex corrections. In the limit of small q^2 we expand[7]

$$\mathcal{A}(q) = \frac{a_0}{q^2} + \frac{a_1}{\sqrt{|q^2|}} + a_2 \ln(|q^2|) + \mathcal{A}_{analytic}(q^2)\,. \tag{1.22}$$

The analytic contributions, $\mathcal{A}_{analytic}(q^2)$, *i.e.,* which can be expanded in non-negative integer powers of q^2, are divergent and must be absorbed in the renormalization of the constants c_i. Thus they cannot be evaluated within the pure Einstein-Hilbert theory. These terms dominate the large-momentum contributions to the loop integral and thus naturally belong in the high-energy regime.

The non-analytic terms $\sqrt{|q^2|}$ and $\ln(|q^2|)$ are due to graviton modes with soft momenta $q^2 \to 0$ running in the loop, and they give non-contact, power-law corrections r^{-n} to the potential $V(r)$. They are finite, inambiguous and fully calculable using the Einstein-Hilbert theory. The contribution from logarithms is[8]

$$\delta V(r) = -\frac{41}{10\pi}\frac{Gm_1m_2}{r}\frac{G\hbar}{r^2} \tag{1.23}$$

which explicitly involves the Planck length (squared) $G\hbar$.[9]

The conclusion is that GR is a good quantum effective theory, with (limited) predictive power and calculable quantum corrections at energies below the Planck scale. Indeed, it is the best quantum field theory for the real world that we have, in the sense that typically other theories cease to be valid much below the Planck scale, and all of them necessarily get corrections at this scale through their coupling to the gravitational field.

[7]When computing the static interaction potential one takes spatial momenta $q^2 = +\mathbf{q}^2 \geq 0$.

[8]The term $\sqrt{|q^2|}$ gives a classical non-linear correction to Newtonian gravity.

[9]The correction $\delta V(r)$ is related to, but not the same as, the correction to metric coefficients such as g_{tt}.

2 Breakdown of the effective theory

Growth of gravitational interaction. The perturbative effective theory of GR breaks down at energies $E \sim M_{Planck}$. One way to see this is that scattering amplitudes grow too large. The theory is strongly coupled. Perturbative unitarity bounds are violated.[10] The reason is that gravitons couple to energy, and therefore the scattering amplitudes grow unbounded at very high energies. In fact, gravity becomes the dominant interaction at Planckian energies. In more detail, consider the '$2 \to 2$' scattering at large center-of-mass energy, when mediated by exchange of particles of different spin[11]:

- For scalar interactions (*e.g.,* ϕ^3), the vertices do not depend on energy, and thus the scattering amplitude behaves like $A_0(s,t) \sim s^0/(-t)$.
- Vectors A_μ couple to currents (*i.e.,* to velocities) so their vertices carry factors of $p^\mu \sim \sqrt{s}$, and thus scattering amplitudes grow like $A_1(s,t) \sim s/(-t)$.
- Tensors $h_{\mu\nu}$ (gravitons) couple to stress-energy $p^\mu p^\nu \sim s$ (this is visible in the two-derivative vertex (1.5)), so the scattering amplituds grows like $A_2(s,t) \sim s^2/(-t)$,[12] and the cross section like $\sigma \sim s^2$.[13]

In short: the effective dimensionless coupling for gravity, Gs, runs with the energy as a power and eventually overwhelms any other interaction. Gravity becomes strongly coupled when $Gs > 1$, and the perturbative expansion in the effective theory of quantum GR is no longer sensible.

One manifestation of this is that gravitational scattering amplitudes grow too large to be compatible with unitarity[14]. When $s \gg |t|$ *i.e.,* forward scattering, unitarity can be recovered by a resummation to all orders of the leading powers in s of a class of diagrams (eikonal approximation). This describes scattering via a large number of exchanged quanta, each of which is very soft, with individual momentum exchange

$$q^2 \sim \frac{\hbar}{G}\frac{t}{s}\,. \tag{2.24}$$

The impact parameter is $\Delta x = \hbar/q \gg L_{Planck}$, and therefore one does not probe the regime of very short distances. To leading order in s these processes are effectively classical (the scattering particles remain onshell throughout and no virtuality runs in the loops), describing small angle scattering ($\theta = 4G\sqrt{s}/\Delta x$) which can be accounted for within the Einstein-Hilbert theory.

[10] Perturbative unitarity: writing the S-matrix as $S = I + iT$, unitarity $SS^\dagger = 1$ requires $-i(T - T^\dagger) = T^\dagger T$. Thus, if T is expanded perturbatively, the second-order contribution to $T - T^\dagger$ must be determined by the first-order result for T inserted in $T^\dagger T$. In a perturbative expansion in which higher-order contributions should become smaller, this implies that contributions at each order cannot be too large — roughly, $|T| \lesssim O(1)$.

[11] We characterize it with the Mandelstam kinematic invariants, $s = -(p_1 + p_2)^2$ for the center of mass energy squared, $t = -(p_1 - p_3)^2$ for (minus) the square of the exchanged momentum.

[12] In general the interaction mediated by a particle of spin J gives $A_J(s,t) \sim s^J/(-t)$ at high energies.

[13] $d\sigma/dt = |A(s,t)|^2/(16\pi s^2) \sim s^2/t^2$. Then integrate over t with a suitable cutoff $\sim E^2_{Planck}$.

[14] Froissart bound: $\sigma_{tot} < \frac{4\pi}{m^2_{min}} \ln^2(s/s_0)$, but this does not really apply here since the exchanged particle is massless ($m_{min} \to 0$). One must study unitarity in partial-wave amplitudes, $|a_l(s)| \leq 1$.

The real difficulties arise when all kinematic invariants are around the Planck scale, $s \sim |t| \sim M_{Pl}^2$, and the scattering is hard, dominated by the exchange of one quantum (or a few) of Planckian energy, which probes distances $\Delta x \sim 1/\sqrt{|t|} \sim L_{Planck}$. This is the regime of fixed angle, deep inelastic scattering. In this case the perturbative expansion breaks down badly, and there is no known way to resum diagrams to recover good behavior.

Enter new UV degrees of freedom? The conventional QFT wisdom says that at this characteristic scale new degrees of freedom must enter that unitarize the theory. For instance, the Fermi theory of weak interactions remains very good at energies $\lesssim O(100 \text{ GeV})$ but the scattering amplitudes violate unitarity above this energy. Then one resorts to the electroweak theory which resolves the four-fermion, current-current interaction vertex into a 2 fermion-2 fermion interaction mediated by W and Z gauge bosons with masses $O(100 \text{ GeV})$. In this case we introduce new degrees of freedom in order to *resolve the interactions*[15] but the other degrees of freedom (leptons, quarks, photons) are still part of the high-energy theory. In other instances *the low-energy degrees of freedom are themselves resolved* into more fundamental ones, and thus the interactions are smeared, too. This is the case for QCD, for which at energies below the chiral-symmetry-breaking scale the physics is described using a non-linear sigma model for pions, while at high energies these are seen to be quark-antiquark bound states. The fundamental theory involves variables (quarks and gluons) that are invisible at low energies. Observe that in this case, similarly to the case of gravity, it is the derivative nature of the couplings of (Goldstone) pions that brings in the bad high energy behavior.

Thus one may expect that gravitons, or at least their interactions, are modified by the appearance of new, short-distance degrees of freedom. Indeed, string theory would seem to realize this goal by resolving *both the particles and their interactions* into extended string-like objects that merge and split forming continuous worldsheets. Actually, if the coupling of closed strings is weak, $g_s \ll 1$, they modify the scattering even before the Planck scale is reached. The stringy nature of particles begins to be visible (in the form of low-lying massive excitations of the string) at a scale $M_{string} \sim g_s M_{Planck} \ll M_{Planck}$.

In other alternatives, the gravitational field is replaced by some other structure that may not involve continuous field-theoretical degrees of freedom, as in loop quantum gravity. Finally, one can also contemplate the possibility that the problems mentioned above are an artifact of the perturbative expansion, while General Relativity, augmented with a *finite* number of higher-order terms, makes sense as a quantum field theory non-perturbatively, by flowing to a non-Gaussian fixed point at asymptotically high energies.

Our arguments below, however, provide evidence that such descriptions based on degrees of freedom roughly localized at points, possibly smeared on scales $\sim L_{Planck}$, for which the total number of degrees of freedom scales like the volume, are incorrect. In fact, as we will comment

[15] *i.e.,* replace the current-current vertex G_F with a massive-particle exchange $g^2/(p^2+M_W^2)$, with $G_F \sim g^2/M_W^2$, which suppresses the interaction a high momenta. This yields behavior in accord with the one discussed above for vector-exchange.

later, string theory already hinted at this.

3 Trans-Planckian regime: black hole dominance

In the effective field theory view discussed above, GR is adequate for describing scattering at sub-Planckian energies, but appears to be definitely invalidated when we try to go to $E \gg M_{Planck}$. At these scales, the conventional story goes, the outcome of the scattering is totally incalculable without using the more fundamental microscopic theory. This is in fact what is often meant by saying that "we don't know anything about quantum gravity".

These arguments, however, were perturbative in nature. GR is a highly non-linear theory and we have a good understanding of some crucial non-perturbative effects. Indeed, these become important at high energies even at the classical level. As we will see, this modifies drastically the picture.

Black hole formation in trans-Planckian collisions. Let us reconsider the scattering of two particles in the regime where $s \sim |t| \gg M_{Planck}^2$. There are two different length scales that characterize this problem: one is the classical gravitational radius[16]

$$R_G \sim G\sqrt{s} \tag{3.25}$$

and the other is the quantum uncertainty scale

$$b \sim \frac{\hbar}{\sqrt{|t|}}\,. \tag{3.26}$$

They correspond, respectively, to the Schwarzschild radius of a black hole of mass $\sqrt{s}$, and to the impact parameter of the collision. In the trans-Planckian regime where $s \sim |t| \gg \hbar/G$,

$$R_G \gg b\,. \tag{3.27}$$

The picture is this: we are trying to concentrate a large amount of energy into a small region of size $\sim b$. Classical GR tells us that gravitational collapse will occur, with the formation of a black hole of size $\sim R_G$. The black hole horizon cloaks the physics in the region of size b that we were expecting to investigate in our hard-scattering experiment. This horizon effects the *causal disconnection* of the outside observer from the region of infra-Planckian distances. Thus, the result of the experiment will be insensitive to this physics.

In more detail. The argument for black hole formation can be developed in some more detail:

- The collision of two ultra-relativistic particles has been examined in a variety of manners: in the infinite boost limit, the gravitational field of a particle becomes a gravitational

[16]For simplicity of presentation we make our arguments in a four-dimensional setting, where $M_{Planck}^2 = \hbar/G$, $L_{Planck}^2 = \hbar G$, but this can be easily extended to any $D \geq 4$.

(Aichelburg-Sexl) shock-wave. By causality, one can construct a solution by superimposing two of these shockwaves, heading towards each other with finite impact parameter, up to the moment when the shocks meets. Before this happens, compact trapped surfaces appear, bounded by an apparent horizon with an area that is indeed $\sim R_G^2$. The evolution past the point where the shocks meet requires dealing with the full complex non-linearity of Einstein's equations. Nevertheless, classical singularity theorems show that after the appearance of compact trapped surfaces, a singularity will appear in finite time, and if cosmic censorship holds, this singularity will be cloaked by an event horizon with an area not smaller than the area of the apparent horizons.

- Recently, full numerical simulations of ultrarelativistic collisions have been carried out which confirm this picture (although, so far, only at zero impact parameter). After the emission of gravitational radiation (a fraction of several percent of the total energy) the system does settle down to a black hole.

One may be worried that these studies of scattering completely ignore the quantum nature of the colliding particles. Indeed, in the numerical simulations the colliding objects are classical 'scalar stars' or vacuum black holes, which do not resemble at all the intrinsically quantum particles in a high-energy experiment. Moreover, since classical (vacuum) GR does not have any scale, the process cannot be properly characterized as trans-Planckian, but only as ultra-relativistic, and obviously it cannot reveal anything about Planck-scale physics.

The point is, though, that whether the *source* of the gravitational field is quantum or classical does not matter, since *the collision is not so much among the particles as among the classical gravitational fields they create—e.g.,* in the first example above, it is the gravitational shock-waves far from their center that are involved in the formation of the trapped surface. These gravitational fields are *classical* at large distances away from the particle source, in particular in the regions where the apparent horizon appears.[17] They are coherent states of the field involving many gravitons. Thus it is not adequate to view the collisions as occurring between a few hard quanta, but rather between a large number of long-wavelength, soft quanta.

It is thus remarkable that in order to describe this process, only classical GR is needed. In fact, classical GR becomes better the larger the energy is: at large s the classical radius R_G is big, and the curvatures in the region where the horizon forms are small. Super-Planckian curvatures (which to long-wavelength observers are, effectively, singularities) are restricted to a region that lies deeper and deeper inside the black hole.

[17] There is one subtle caveat. In the case where the collision involves a fully localized (pointlike) quantum object, some studies seem to suggest that bremsstrahlung may give rise to emission of a few very energetic gravitons at scales parametrically larger than R_G, *i.e.,* before the apparent horizon forms. At present the situation is confusing, and even if this effect is actually there it is unclear whether it would impede the creation of the black hole — note that the radiation would be expected to be highly collimated in the forward direction, and thus may after all fall into the black hole. And even if it hindered black hole formation, it would also prevent probing the trans-Planckian regime.

Black hole decay. In a quantum theory, the black hole will not be the absolute stable endpoint of this scattering event. The black hole will decay by Hawking emission of quanta with energies

$$\omega \sim T_H \sim \frac{1}{R_G} \,. \tag{3.28}$$

These will have large multiplicities

$$N \sim \frac{M}{\omega} \sim \frac{M}{T_H} \sim S_{BH} \sim \left(\frac{M}{M_{Planck}}\right)^2 \sim \frac{s}{M_{Planck}^2} \gg 1 \,. \tag{3.29}$$

Thus, the experimenter will not be realizing a hard '2 $\to$ 2' scattering: this involves a huge reduction in entropy and the amplitude is suppressed by a factor $e^{-S_{BH}}$. Instead, one gets a soft '2 $\to$many' scattering in which only distances of order R_G are probed. Note that the problems with perturbative unitarity disappear, since the unitarity-violating hard scattering amplitudes have been exponentially suppressed.

The outcome of the experiment is therefore dominated by semiclassical GR physics: the formation and subsequent slow Hawking evaporation of a black hole. Remarkably, *both the sub-Planckian and the trans-Planckian regimes of scattering can be described using the semiclassical physics of Einstein's theory.* In the sub-Planckian regime the theory is perturbative, and in the trans-Planckian regime it is non-perturbative, but the further we are from the Planck scale, in each direction, the better the classical theory becomes.

3.1 A 'generalized uncertainty principle'

It is sometimes suggested that the appearance of the two scales (3.25) and (3.26) in attempts to probe short distances is a hint of a 'generalized uncertainty principle' (GUP)

$$\Delta x \gtrsim \frac{\hbar}{E} + GE \tag{3.30}$$

that incorporates Heisenberg's quantum uncertainty, which forbids the exploration of regimes where $\Delta x < \hbar/E$, and the causal limitations from classical black hole horizons, which prevent attempts at probing $\Delta x < GE$. Thus there seems to be a fundamental lower limit at $\Delta x \simeq \sqrt{\hbar G} = L_{Planck}$. However, this GUP is no more than a fancy way of stating these latter facts. There is at present no derivation of it from any fundamental theory, and one should be cautious when trying to extract further consequences from it. In particular, it is unclear if, and in what sense, the quantum theory of gravity must contain a minimum length as part of its fundamental definition. We will see that this does not happen in AdS/CFT.

4 Semiclassical Quantum General Relativity is not enough

We have been arguing that Einstein's theory allows us to compute the outcome of scattering experiments performed at both sub-Planckian and trans-Planckian energies. The scattering amplitudes thus computed do not violate the perturbative unitarity bounds in neither of those regimes. Does this mean that we need not look any further for a UV-complete theory of gravity? Seems the answer is no.

Interpolating between sub-Planckian and trans-Planckian. In the first place, there is the regime of energies around the Planck scale in which the effective description fails, and in which black holes (and other gravitational fields) are not appropriately described as semiclassical objects. In order to interpolate between the sub-Planckian and trans-Planckian regimes we certainly need physics not contained within GR.

String correspondence to black holes. Our best candidate for this interpolation is string theory. As we have mentioned above, when the string coupling g_s is small, string-theory effects begin to manifest before the Planck scale is reached. At $E \sim M_{string} \sim g_s M_{Planck}$ the first string resonances can be excited. As their mass grows and we get to highly-excited string states, with $M \gg M_{string}$, two effects appear:

- The degeneracy of string states of a given mass grows exponentially, $\rho(M) \sim e^{M/M_{string}}$. These are unstable and decay by emitting radiation[18] at temperature $T \sim M_{string}$.
- The size of the typical string state grows much larger than the string length $\ell_{string} \sim M_{string}^{-1}$. The string tends to spread like a random-walk[19], with size $R \sim \sqrt{M/M_{string}}\,\ell_{string}$. However, as the mass grows, the gravitational self-interaction of the string makes it more compact than this size.

Thus the string becomes a highly entropic object ($S_{string} \propto M$). If we put more energy into a string, trying to probe shorter scales, we find instead a softer object with a size that grows with its energy. Thus perturbative strings already exhibit the difficulties in probing ultra-short distances, and in fact a stringy GUP has been formulated

$$\Delta x \gtrsim \frac{\hbar}{E} + \alpha' E \tag{4.31}$$

in which $\ell_{string} = \sqrt{\hbar\alpha'}$ replaces L_{Planck} as the minimum length scale that can be probed.[20]

These are also features of black holes, which suggests that massive string states may perform the interpolation between the semiclassical perturbative and non-perturbative regimes of GR.

There is an apparent problem in that neither the growth with M of the entropy nor the size are exactly like those of a black hole. However, we do not need to find agreement at all values of M, but only in a small range of energies that may be dubbed the *'black hole/string correspondence point'*. In this range, we would expect to see that the string smoothly morphs into an object better described as a black hole. There is indeed good evidence that when the mass of the string reaches $M \sim M_{string}/g_s^2$, the entropy, temperature, size[21], and all other properties of the string are parametrically like those of a black hole of the same mass.

[18]The picture can get somewhat more complicated since long strings can also break into two long strings. Nevertheless, at finite coupling and large masses one expects that the gravitational attraction will hold together the system against such splittings.

[19]The energy of the string is proportional to its total length L, *i.e.,* $M/M_{string} \sim L/\ell_{string}$. The size of a random walk of total length L is well-known to scale like $R \propto \sqrt{L}$.

[20]Note that in perturbative string theory $\ell_{string} \sim L_{Planck}/g_s \gg L_{Planck}$.

[21]As mentioned above, self-gravitational effects are needed for this.

For strongly coupled strings $g_s \sim O(1)$, the correspondence occurs at the Planck scale and it is unclear how to use string theory[22] to describe the connection between the sub- and trans-Planckian regimes.

Black hole unitarity and trans-Planckian physics. A more subtle but much more serious and unusual problem is caused by the fact that, even if black hole formation saves perturbative unitarity at trans-Planckian energies, this does not automatically imply that the entire process is governed by unitary evolution.

In fact, in the previous lecture we argued that the framework of local (effective) quantum field theory, including GR, leads to the puzzle of apparent loss of unitarity in the evolution of black holes. This is a very subtle effect, which does not seem to be visible in studies of perturbative unitarity[23]. The preferred resolution (for us) was that unitarity is preserved at the cost of introducing highly non-local effects, which would affect the process of Hawking radiation at the horizon, *i.e.,* at scales that can be much longer than the Planck length, in regions of arbitrarily small curvatures. Such long-wavelength modifications of semiclassical GR seem to point to the need for a departure from the conventional ideas about the decoupling of different energy scales in quantum field theory[24].

5 A Quantum theory of Gravity is not a local QFT

Quasi-locality of observables in General Relativity. To be elaborated on.

Wilsonian QFT. Conventional (textbook, Lagrangian) quantum field theories are based on the idea that one can separate the propagation of the field from its interactions. The propagation is controlled by a free theory, and this is what dominates the short-distance structure of the field[25]. Interactions are then added on as perturbations. The intuition is that the coupling constant is a measure of the rate of interactions per unit time. At very high frequencies, these interactions are too rare during an oscillation of the field, but as the frequency is lowered, they become more relevant.[26] Wilson generalized this framework to include theories in which the short-distance structure need not be dominated by a free theory, but rather one in which interactions may be strong. If there is a good limit to the theory at asymptotically high energies, this should be a Conformal Field Theory (CFT), *i.e.,* one that is invariant under (local) changes of the energy scale[27]. The analogue of the perturbative interactions of a free theory is now

[22] This is, using worldsheet variables instead of recurring to the holographic CFT dual, to be discussed below.

[23] In fact, even if mini-black holes were copiously produced in colliders, it seems extremely difficult to determine experimentally whether this non-perturbative violation of unitarity occurs.

[24] Which involve locality in energy of the renormalization group equations.

[25] Since these involve path integrals with actions quadratic in the fields, one refers to free theories as Gaussian.

[26] The knowledgeable reader no doubt realizes that here 'relevant' is a technical term. Marginally relevant couplings, as in QCD, are of course a bit subtler.

[27] I will not enter in the discussion of under what circumstances scale invariance implies conformal invariance.

played by perturbations of the CFT by operators that may become important (relevant) at lower energies, but whose effects do not grow, or more typically die off, at higher energies.

Locality plays a crucial role here. One assumes that in order to define the CFT, one has to look at the short-distance structure of the theory. It is there where the fundamental degrees of freedom are manifest. The long distance features of the *state* of the field are not fundamental, but rather properties of *specific solutions* of the theory.

One consequence of this locality is that the total number of degrees of freedom should be an extensive quantity, growing like the volume of the region one is considering. Take the theory to be defined on a spatial sphere S^{D-1} of radius R. In order to see how the spectrum of states grows with the energy, we can imagine putting the system at temperature T, and measuring its total entropy and energy. Since the entropy is dimensionless but must scale like the spatial volume, dimensional analysis dictates that

$$S_{CFT} = a(RT)^{D-1}\,, \tag{5.32}$$

and thus the energy

$$E = a\frac{D-1}{D}R^{D-1}T^{D}\,. \tag{5.33}$$

Then

$$S_{CFT} = c(ER)^{\frac{D-1}{D}} \tag{5.34}$$

The number c (and a, which is fixed by it) is theory-dependent, and provides a measure of the number of local fields (species of degrees of freedom) in the theory.

CFTs are then at the heart of the fundamental definition of QFTs. The question is now whether GR can be formulated as a QFT in this sense. We argued above that the perturbative non-renormalizability of GR indicates that this cannot be realized in terms of a Gaussian, free theory with a finite number of interactions. Still, one may entertain the possibility of a CFT that captures a non-Gaussian fixed-point at high energies for the Einstein-Hilbert theory augmented with a finite number of higher-order terms. This would be a non-perturbative definition of the theory based on gravitational degrees of freedom.

From our considerations above, the possibility that a QFT of gravity can be defined in this way would seem to require that

> the entropy that characterizes the density of states in a region of size R should scale with energy like (5.34), independently of what the state of the field is at long distances.

In addition, since the CFT fixed point dominates physics when all kinematic invariants are large, its asymptotic scale invariance dictates that at very high energies

> correlation functions should depend only on powers of ratios of the ingoing and outgoing momenta.

Non-Wilsonian character of quantum gravity. *What we know about semiclassical black holes tells us that neither of these two properties is satisfied by a quantum theory of gravity.* Already the thought scattering experiments that we discussed above have revealed that scattering amplitudes are not hard: for '$2 \to 2$' scattering, the amplitudes never become power laws no matter how high the scattering energy, but are instead exponentially suppressed by $e^{-S_{BH}} \sim e^{-E^2/E^2_{Planck}}$.[28] The probabilities to emit quanta of a given energy ω are $\sim e^{-\omega E/E^2_{Planck}}$.

Moreover the long-distance properties of the spacetime are crucial to the outcome of the scattering event: at higher and higher energies, the black hole that one forms gets larger and larger. Thus, how its area (entropy) grows with mass, and how the black hole decays, are going to be sensitive to what the spacetime is like at large length scales.

If we try to localize energy into a region of size R, we know that in GR eventually we will form a black hole. If the spacetime is asymptotically flat, then the density of states that we get does not scale like (an exponential of) the volume R^{D-1}, but rather like the area R^{D-2}. Thus we will not obtain (5.34). Instead, the Schwarzschild-type black hole that one creates has

$$S_{BH} = b\left(\frac{E}{E_{Planck}}\right)^{\frac{D-2}{D-3}} \tag{5.35}$$

(again b is a pure number). The different scaling with energy indicates that the asymptotic spectrum of states is not that of a CFT.

In these two aspects the origin of the difference between the CFT behavior and the behavior of the black hole in pure gravity is that the size of quantum effects in the latter are always measured by the ratio of the energy to the Planck scale, E/E_{Planck}. Thus, the fundamental scale never disappears, and scale invariance is never recovered.

6 Quantum gravity is holographic

The two main lessons we obtain from this are:

- One cannot separate the asymptotic behavior of the spacetime from the properties of the spectrum of fundamental degrees of freedom. Long-distance physics enters the fundamental definition of the theory at the highest energies (UV/IR connection).

- The number of degrees of freedom needed to fully describe a region of radius R grows like the area $\sim R^{D-2}$ of the boundary of that region.

It is then natural to suggest that quantum theories of gravity must be formulated in terms of degrees of freedom localized on surfaces, and not at points (or over regions of Planckian size): GR is not a local QFT, but instead a *holographic* theory.

Moreover, the low-energy effective QFT of gravity is deceptive, in that a seemingly low-energy, long-distance parameter like the cosmological constant, which one would think of as an

[28] In string theory, hard, fixed-angle scattering is suppressed, at n-loop order, as $e^{-sf(\theta)/(nM^2_{string})}$.

effective one that receives contributions from physics at all scales, is actually a fundamental parameter that enters the microscopic definition of the theory.

7 Gravity in AdS: Towards AdS/CFT

The result (5.35) refers to a black hole in asymptotically flat space. However, as we have argued, the asymptotic density of states in a quantum theory of gravity will depend on the value of the cosmological constant.

Static black holes in anti-deSitter spacetime are described by

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_{D-2}\,, \tag{7.36}$$

with

$$V(r) = 1 - \frac{\mu}{r^{D-3}} + \frac{r^2}{L_{AdS}^2} \tag{7.37}$$

where μ is the mass parameter, $M \propto \mu/G$, and $L_{AdS} \propto 1/\sqrt{-\Lambda}$ is the cosmological radius. Black hole horizons are located at radii where $V(r) = 0$. When $\mu \ll L_{AdS}^{D-3}$ the horizon radius is $r_H \simeq \mu^{1/(D-3)}$, much smaller than L_{AdS} and the black hole hardly feels the presence of the cosmological constant: it is much like the Schwarzschild solution in AF space.

However, we are more interested in what happens when the energy is large, $\mu \gg L_{AdS}^{D-3}$. In this case[29] we can neglect the constant '1' in $V(r)$ and find the horizon from

$$\frac{\mu}{r_H^{D-3}} \simeq \frac{r_H^2}{L_{AdS}^2} \tag{7.38}$$

i.e.,

$$r_H^{D-1} \simeq \mu L_{AdS}^2 \sim GEL_{AdS}^2\,, \tag{7.39}$$

or[30]

$$\left(\frac{r_H}{L_{Planck}}\right)^{D-1} \sim \frac{L_{AdS}}{L_{Planck}} EL_{AdS}\,. \tag{7.40}$$

The entropy is

$$S_{BH} = \frac{\Omega_{D-2}}{4}\left(\frac{r_H}{L_{Planck}}\right)^{D-2} \sim \left(\frac{L_{AdS}}{L_{Planck}}\right)^{\frac{D-2}{D-1}} (EL_{AdS})^{\frac{D-2}{D-1}}\,. \tag{7.41}$$

If we compare this to (5.34) we see that the scaling with the energy differs from that of a CFT in D spacetime dimensions. However, it is the same as that of a $(D-1)$-dimensional CFT on a

[29] If we take the temperature as our control parameter, and attempt to study the theory at high temperatures, we find that the small black hole, with $T \sim 1/r_H$, is indeed a possible solution. However, the large black hole with the same temperature (now with $T \sim r_H/L_{AdS}^2$) has much higher entropy — much lower free energy — and dominates the canonical ensemble. Moreover, it is not only globally but also locally thermodynamically stable.

[30] Recall $G\hbar = L_{Planck}^{D-2}$.

sphere of radius $R \sim L_{AdS}$, with[31]

$$c \sim \left(\frac{L_{AdS}}{L_{Planck}}\right)^{\frac{D-2}{D-1}} . \tag{7.42}$$

The conclusion is that AdS$_D$ gravity may admit a description in terms of a CFT$_{D-1}$ with a large number of fields of the order of c in (7.42). This must be large if the curvature radius of the spacetime L_{AdS} is to be much larger than the Planck length, which is required for the semiclassical validity of the GR description. Observe that this number of fields does depend on the cosmological constant. So, in the fundamental theory, the latter will be quantized, and moreover, different values of Λ will correspond to different theories.

7.1 AdS/CFT realized

The previous ideas have been realized in several cases. The most famous one is the AdS$_5$/CFT$_4$ correspondence between the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with group $SU(N)$ and type IIB supergravity in AdS$_5 \times S^5$. The S^5 in the latter is a compact space and does not add to the dimensionality of the boundary, but instead realizes the conformal scalar fields of the Yang-Mills theory. The number of local fields is $c \sim N^2$ and thus the rank of the gauge group must be large in order for the gravitational description to be valid. One can in fact go beyond the gravitational regime in the AdS side of the correspondence, and consider type IIB string theory in this spacetime.

Eq. (7.42), with $D = 5$, contains the 5D Planck length $L_{Planck(5)}$. This is related to the 10D Planck length in AdS$_5 \times S^5$ as

$$\left(L_{Planck(10)}\right)^8 = \left(L_{Planck(5)}\right)^3 \left(L_{S^5}\right)^5 = \left(L_{Planck(5)}\right)^3 \left(L_{AdS}\right)^5 \tag{7.43}$$

so

$$c \sim \left(\frac{L_{AdS}}{L_{Planck(5)}}\right)^{3/4} \sim \left(\frac{L_{AdS}}{L_{Planck(10)}}\right)^{2} . \tag{7.44}$$

Since $c \sim N^2$, the AdS radius measured in Planck units is

$$\frac{L_{AdS}}{L_{Planck(10)}} \sim N^{1/4} . \tag{7.45}$$

Generically, a holographic theory need not have any more dimensionless parameters than c, but the SYM theory does have another one: the gauge coupling constant (a continuous modulus) g_{YM}, or the 't Hooft coupling $\lambda = g_{YM}^2 N$. This introduces another parameter in the gravitational side, namely the string coupling constant

$$g_s \sim g_{YM}^2 \sim \frac{\lambda}{N} , \tag{7.46}$$

[31]Observe that this factor is extracted after factorizing out all the dependence on the energy. The microscopic Planck scale, which could not be eliminated from (5.35), has now been transferred to the parameter c via the introduction of a macroscopic scale L_{AdS}.

and since $L^8_{Planck(10)} \sim g_s^2 \ell^8_{string}$,

$$\frac{L_{AdS}}{\ell_{string}} \sim \lambda^{1/4} \,. \tag{7.47}$$

String effects are small for large λ.

7.2 Some remarks on the correspondence

Planck and string scales from a scale-free CFT. The parameters for the microscopic theory are only N and g_{YM}. The Planck length has not in any way been introduced as a fundamental minimal length. Neither has the string length. So this quantum theory of gravity *does not contain any dimensionful parameter, but instead has two dimensionless parameters.* It is interesting to contrast this to the emphasis often made on the seemingly appealing fact that string theory does not contain any fundamental dimensionless adjustable parameters, but only has a dimensionful one (the string tension). In the quantum theory of gravity in AdS the absence of a fundamental parameter for the Planck scale is possible because the theory has a cosmological constant.

This brings us to a point that could have been raised many years ago about how, or even whether, a string description of certain Yang-Mills theories is possible at all.[32] QCD has string-like excitations corresponding to color-flux tubes stretching between quarks. These strings have a tension, with a scale that in QCD is naturally associated to the dynamically-generated mass-gap scale Λ_{QCD}. Thus, even if the theory is classically scale invariant, through quantum effects it is possible to have strings and massive excitations.

Is this possible if the YM theory is exactly scale-invariant? That is, how can strings, with a non-zero string scale tension, appear in an exact, scale-free CFT such as $\mathcal{N} = 4$ SYM? The crucial point is that the strings propagate in a spacetime different than the one in which the CFT is defined, in fact a spacetime with a characteristic length scale. The string scale, just like the Planck scale discussed in the previous paragraph, exists *only* with reference to this background scale.

In other words, the CFT only contains pure numbers, not scales. If these pure numbers (*e.g.,* dimensions of operators) are to be translated into magnitudes of a dual theory that necessarily involves dimensionful scales, such as a string theory or a quantum gravity theory, then we conclude that the latter theories must be defined in a spacetime that contains a length scale. We choose N and g_{YM} to be large in order to have large parametric separations in the mass spectrum of the dual theories.

Cosmological constant problem. Notice also that the cosmological constant is naturally quantized in Planck units, and from the gauge theory viewpoint there is no reason why it should run to Planckian values. It is small simply because we take N to be large. Thus the dual gauge

[32] This could have been raised ever since $\mathcal{N} = 4$ SYM was argued to be an exact CFT. I am not aware of any explicit mention of this problem in the pre-AdS/CFT literature, although I suspect it must have been realized by some.

theory appears to solve the cosmological constant problem.[33] However, in analogy with the way it also resolves the black hole unitarity problem, it provides little understanding on the bulk viewpoint of the resolution.

One may invoke the large amount of supersymmetry in IIB supergravity. However, the gauge theory argument — namely, that L_{AdS}/L_{Planck} is not an effective but a microscopic parameter of the theory — does not rely on supersymmetry in any obvious way. Supersymmetry may however be necessary (or at least very helpful) for stabilizing the large hierarchy between the few operators with low conformal dimension that are dual to the light supergravity fields, and the rest of the CFT spectrum with much larger dimension. This "unnatural" hierarchy is necessary for a good gravitational theory of the bulk, and a large amount of symmetry may be the way to stabilize it.

Other correspondences, more quantum gravities. One can deform the SYM theory by adding relevant (or marginal) operators. Generically the dual gravitational theories do not have AdS as a solution now, but nevertheless they are still asymptotically AdS. They have been constructed in many cases, typically in approximate analytical or numerical form.

Other examples of interest which are under good control occur as AdS_3/CFT_2 correspondences, which are excellent laboratories for the microscopic analysis of black holes, and also AdS_4/CFT_3 theories in which the CFT is a Chern-Simons theory coupled to conformal matter.

String theory has also provided other quantum formulations of gravitational theories that are presumably complete, and which are also holographic in nature: M(atrix) theory, little string theories, and dualities to theories of membranes or fivebranes. All these are, to varying degrees, less well understood than the previous ones.

Matrix quantum mechanics is very interesting since it may provide large classes of quantum theories of gravity that defy traditional assumptions. In the large N limit, these models present some features of gravity and black holes (large degeneracies, scrambling and chaos). If the matrices are endowed with a vector index with symmetry $SO(d)$, then they may yield a quantum theory of gravity in $d+1$ or $d+2$ dimensions. Supersymmetry does not seem to be required for finiteness, which is automatic because the theory is simply quantum mechanics. So there appears the possibility that finite quantum gravity exists in any dimension, and without supersymmetry.

Before reaching this conclusion, we must bear in mind that there are at least two other conditions that seem necessary to claim that we have a good holographic quantum theory of gravity: stability (boundedness-below) of the spectrum, and a large hierarchy between a few operators of low dimension and other operators of large dimension. For these two purposes, supersymmetry may be very helpful, but it is unclear whether it is indispensable.

[33] If $AdS_5 \times S^5$ is viewed as the near-horizon limit of N D3-branes, then it is also clear why N is an integer that is not renormalized.

8 Quantum theories of gravity, the role of string theory, and alternatives

8.1 Holographic quantum theories of gravity

According to the arguments we have developed, quantum theories of gravity are holographic in nature: they are not specified in terms of local degrees of freedom in the spacetime where gravity lives. Instead they are defined by quantum theories that live in the boundary of spacetime.

Then, it does not seem appropriate to talk about 'the theory of quantum gravity' as if this were a unique theory that characterizes the microscopic gravitational degrees of freedom and their interactions, once and for all, independently of what the state of the field is at long distances. Instead, spacetimes with different asymptotics will correspond to different quantum theories of gravity, with microscopic degrees of freedom that may be completely different in each case. The picture is one in which gravity 'emerges' as a semiclassical description of the dynamics of certain classes of quantum theories. Many different quantum theories may admit such dual gravitational descriptions, so we speak of 'quantum theories (or models) of gravity'.

In this picture gravity is definitely not a fundamental interaction. Gravitons appear only as collective excitations valid for describing quantum perturbations of the effective geometry with wavelength $\gg L_{Planck}$.

In some regimes, *e.g.,* near the end of black hole evaporation, or close to singularities, the gravitational description will break down, but then the dual quantum theory should provide the correct physics. It is a big challenge to answer these questions, but at least the explicit realizations of AdS/CFT give a framework where, in principle, they can be posed.

Our examples of holographic theories are for the most part based on local quantum field theories, since these are the quantum theories that we understand best. In this case, the QFTs are controlled by the CFTs at the UV fixed-points, and the gravitational duals are necessarily asymptotic to AdS. We do not have a general understanding of what other asymptotics admit a good quantum holographic dual.

In the case of flat asymptotics, it seems that, if there is a holographic description, the theory cannot straightforwardly be a local QFT. It might be that the arguments can be evaded by having the hologram on a null boundary instead of a timelike one, but this is unclear. Our arguments above also indicate that, given the absence of a cosmological constant, now the microscopic theory must include a length scale, which would seem to preclude that it is a conformal theory: its central charge should be zero or infinity, but not any finite number.[34]

We can obtain asymptotically flat spaces as limits of AdS in which $L_{AdS} \to \infty$, *i.e.,* when we focus on physics occurring at scales much shorter than the AdS radius. However, it is unclear whether one can take this limit in the CFT and recover a sensible quantum theory. In this limit, the central charge would seem to diverge.

[34] In M(atrix) theory this scale can be taken to be the 11D Planck length. Little string theories have a string length.

One can also connect the AdS geometry to an asymptotically flat geometry (in some directions) by considering the spacetimes of D-branes in type II string theories. However, we do not have anything like a complete non-perturbative formulation of these string theories.

From what we have seen, it would seem important to develop a general framework for the formulation of holographic quantum theories of gravity, be them based on local quantum field theories, or on more general quantum theories.

8.2 Are strings fundamental?

Gravitons are massless excitations of closed strings. We have argued that gravitons are not fundamental, so then we may ask what is the role of closed strings. In AdS/CFT, (closed) string theories appear as bulk duals to gauge theories.

There are two possible views regarding the AdS/CFT correspondence. One is that the correspondence is a *duality* between two theories that independently admit complete, non-perturbative formulations, in this case one using field variables (*e.g.,* in terms of a lattice regularization, followed by a continuum limit), and the other in terms of worldsheet variables (more generally, bulk variables). However, this is not actually necessary in order for AdS/CFT to be a valid correspondence that gives us quantum theories of gravity. Indeed, one may question why a given quantum theory of gravity should admit not just one but two independent (and radically different) complete formulations. Some of us are already happy to have one! (i.e., the CFT).

There are strong results that prove that not only gravitons, but in fact many non-trivial features of the spectrum of string theory are reproduced by the gauge theory at strong coupling (there are current attempts at showing that interactions are also correctly reproduced). Thus the gauge theory has a regime in which it correctly captures not only gravity but also perturbative string theory.

The other side of the question is whether a strongly coupled string theory can reproduce the gauge theory at weak ('t Hooft) coupling. The problem is that we do not have any formulation of a worldsheet theory with closed strings, and with high degeneracies at large energies, that is not intrinsically perturbative. For instance, the formulation of closed string field theory involves polynomial actions with an infinite number of terms. This is reminiscent of what happens with the low-energy effective theory of gravitons, and suggests that strings may also be effective.[35]

Thus, a natural, consistent and economical position is to take the CFT as the definition of what one means by the quantum theory of gravity in the bulk, in the cases where a semiclassical description in terms of a gravitational field is adequate.[36] Closed strings would not be more fundamental than the gravitons: they would also be only collective excitations of gauge degrees

[35] Deeper arguments can be found in `hep-th/9411028`.

[36] Besides requiring a large 'number of fields' in the CFT, one also needs (i) that the spectrum contains a reduced number of primary operators with low conformal dimensions, separated by a large gap from all other operators, and (ii) that the correlation functions of these operators factorize, *i.e.,* the theory has a good semiclassical weakly coupled limit. These conditions are satisfied in large N gauge (*i.e.,* matricial) theories (at large λ) but not in, *e.g.,* large N vector theories.

of freedom. In this sense, they would not be more fundamental than the QCD string, or the generic flux-tube strings of Yang-Mills theories – fundamental strings would differ in that they admit a limit where they are very thin and can be consistently quantized perturbatively, but they would still be effective strings.

Whenever there are strong gravitational effects, such as at the endpoint of black hole evaporation, the strings may be of intuitive, even semi-quantitative, help (as in the black hole/strings correspondence) but the full description can only be performed using the gauge theory.

String theory's original claim for a finite quantum theory of gravity came from its good perturbative ultraviolet behavior. It is however unclear what the ultimate import of this is. Besides the known fact that the perturbative series is not convergent but only asymptotic (due to worldsheet instantons), we have already seen a simpler hint that *at any non-zero coupling the high-energy behavior of the theory may drastically change*. The good ultraviolet behavior of string theory crucially depends on the properties of its *infinite* tower of massive states. However, if the coupling g_s is finite, no matter how small, these massive string states will begin, at masses of order M_s/g_s^2, to be better described as black holes rather than as perturbative string states. It is then unclear how these states will contribute to the UV behavior of amplitudes. This ceases to be a fundamental issue if the strings are effective objects and instead the dual holographic gauge theory takes care of the dynamics at finite coupling. It is nevertheless a very interesting and difficult issue.

It would seem, then, that string theory is, on one hand, *a* particularly convenient set of variables for summing the terms at each order in the $1/N$ expansion of gauge theories. On the other hand, strings allow us to use bulk variables (and a bulk description) for gravitational systems in such a way that the description goes beyond what is possible with particles or fields, before entering the regime of black holes. Indeed, it begins to display some of the properties of black holes, in the form of highly-extended and highly degenerate massive string states: *strings are precursors to black holes.*

So, even if they turn out not to be fundamental, strings and string theory are an inherent part of quantum theories of gravity, at least of those with a gauge theory dual, and in all likelihood will continue to play an indispensable role in their investigation.

8.3 Alternatives?

The picture we have arrived at is utterly different from what one often finds in attempts at quantizing gravity. Typically, the presence of the Planck length (commonly given a physical significance through thought experiments such as we have described in the context of the GUP) is regarded as an indication that the short-distance properties of spacetime have to be modified, often by replacing the geometry with some discrete structure that would cut-off the ultraviolet divergences of the theory. The most popular approaches in this class are loop quantum gravity and its spin-foam siblings, but also triangulations of spacetime, causal sets, etc. In other instances, one still tries to retain a continuum field picture, in which the apparent problems with the quantum effective theory of GR are an artifact of the perturbative expansion. Among the lines that currently enjoy some popularity, there are the asymptotic safety scenarios, and theories with broken Lorentz invariance in the UV.

The line of argument we have followed leads to the conclusion that attempts at describing quantum gravity in terms of degrees of freedom that scale extensively with the volume, and in which these degrees of freedom decouple from the long-distance asymptotics of the field, are incompatible with the known properties of black holes.

Holographic behavior requires a drastic reduction in the number of degrees of freedom of the theory. It is claimed that the non-Gaussian fixed point found in some gravitational RG flows does not correspond to a four-dimensional but lower(2)-dimensional theory. Still, this does not match the asymptotic degeneracy in (5.35). In fact, the negative specific heat that this spectrum implies is incompatible with a unitary CFT with a positive c.

Theories where Lorentz invariance is broken in the UV evade our arguments but it remains to be seen whether they admit healthy non-perturbative definitions while reproducing consistent gravitational dynamics in the infrared. The study of black holes and their thermodynamics in these theories should probably throw light on this.

It may be that $\mathcal{N} = 8$ supergravity is a perturbatively-finite theory, but at a non-perturbative level it does not seem to contain the ingredients needed for holography and for a microscopic interpretation of black hole thermodynamics.

I will not say that these studies can not eventually be very interesting and possibly useful for physics. So long as mathematical consistency is preserved, they may teach interesting lessons, for gravity or otherwise — the history of physico-mathematical theories exhibits all sorts of twists and turns. What looks less probable, though, is that a non-holographic quantum theory has a regime where the gravitational dynamics of Einstein's theory is recovered, including its long-lived, highly-entropic black holes and their thermodynamical properties. Any alternative theory of this sort must then explain what went wrong in the arguments and conclusions that black holes have guided us to.