Quantum General Relativity

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It is widely accepted that at energies below the Planck scale one can treat General Relativity as an effective quantum field theory. One can extract those finite quantum-gravitational effects that are due to low-frequency gravitons running in loops, since these are not sensitive to the ultraviolet behavior of the theory.

Perturbative non-renormalizability of GR. Consider the Einstein-Hilbert theory

$$I = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}R\,, \qquad \kappa^2 = 32\pi G \tag{3.1.1}$$

and study small fluctuations around Minkowski space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,. \tag{3.1.2}$$

Expanding in powers of κ,

$$\frac{2}{\kappa^2}\sqrt{-g}R = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \ldots \tag{3.1.3}$$

where, schematically (suppressing Lorentz indices and numerical factors),

$$\mathcal{L}^{(2)} \sim \partial h \partial h \tag{3.1.4}$$

$i.e.,$ terms quadratic in h which yield the propagator of the graviton, while the cubic terms

$$\mathcal{L}^{(3)} \sim \kappa h \partial h \partial h \tag{3.1.5}$$

account for the leading self-interaction of gravitons. Observe that this vertex contains two derivatives, $i.e.,$ two factors of momenta. In the expansion there appear vertices $\mathcal{L}^{(n)}$ involving an arbitrary number of gravitons.

This theory is not renormalizable perturbatively. When one computes loop diagrams, there are divergences that cannot be absorbed by a renormalization of the coupling κ. Veltman and 't Hooft showed that at one loop the divergences (from gravitons only) are

$$\mathcal{L}^{(div)}_{1-loop} = \frac{1}{8\pi^2\epsilon}\left(\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right)\,, \tag{3.1.11}$$

where $\epsilon = 4 - D$ is the parameter in dimensional regularization. Renormalizing this divergence requires the introduction of higher-order terms in the bare action. When expanding around vacuum, $\bar{R}_{\mu\nu} = 0$, this divergence does not appear so the theory is finite at one loop. However, in this case trouble reappears at two loops, where Goroff and Sagnotti found

$$\mathcal{L}^{(div)}_{2-loop} = \frac{209}{2880}\frac{\kappa^2}{16\pi^2\epsilon}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\eta\sigma}R^{\eta\sigma}{}_{\alpha\beta}\,, \tag{3.1.12}$$

which again cannot be renormalized within the original action.

Quantum General Relativity as an effective theory. The theory nevertheless is sensible as a low-energy effective field theory, and can adequately describe scattering processes in which all kinematic invariants remain below the Planck energy, at least within a precision bounded by a power of E/E_{Planck}.

In a low-energy effective theory one considers all terms in the action that are consistent with the symmetries of the system, so[28]

$$I = \int d^4x \sqrt{-g} \left(\Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right) . \tag{3.1.13}$$

The value of the cosmological constant term Λ is a notorious problem that we do not want to deal with now, and we set it to $\Lambda \approx 0$ on observational grounds ($\Lambda \ll E_{Planck}^4$).

Renormalization of the one-loop divergences in (3.1.11) can be achieved by

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2} \frac{1}{\epsilon} , \qquad c_2^{(r)} = c_2 + \frac{7}{160\pi^2} \frac{1}{\epsilon} . \tag{3.1.14}$$